AN ROINN OIDEACHAIS **M. 32**

LEAVING CERTIFICATE EXAMINATION, 1996

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 21 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

1. *(a)* A particle starts from rest and moves in a straight line with uniform acceleration. It passes three points a, b and c where $|ab| = 105$ m and $|bc| = 63$ m. If it takes 6 seconds to travel from a to b and 2 seconds to travel from b to c find

(i) its acceleration

(ii) the distance of a from the starting position.

(b) A lift starts from rest with constant acceleration 4 m/s^2. It then travels with uniform speed and finally comes to rest with constant retardation 4 m/s^2. The total distance travelled is d and the total time taken is t.

(i) Draw a speed-time graph.

(ii) Show that the time for which it travelled with uniform speed is

$$\sqrt{t^2 - d}.$$

2. A ship, B, is travelling due West at 25.6 km/h. A second ship, C, travelling at 32 km/h is first sighted 17 km due North of B. From B the ship C appears to be moving South-east.

Find

(i) the direction in which C is actually moving

(ii) the velocity of C relative to B

(iii) the shortest distance between the ships in the subsequent motion

(iv) the time that elapses, after first sighting, before the ships are again 17 km apart.

OVER→

3. (a) A particle is projected from the ground with a velocity of 50.96 m/s at an angle $\tan^{-1}\frac{5}{12}$ to the horizontal. On its upward path it just passes over a wall 14.7 m high. During its flight it also passes over a second wall 18.375 m high.

Show that the second wall must be not less than 23.52 m and not more than 70.56 m from the first wall.

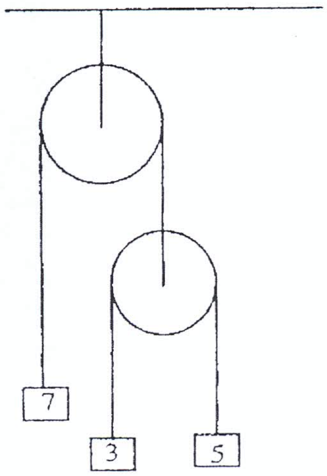
(b) A plane is inclined at an angle of 2β to the vertical. A particle is projected up the plane with initial velocity $u\cos\beta$ at an angle β to the inclined plane. The plane of projection is vertical and contains the line of greatest slope.

Show

(i) that the time of flight of the particle is $\frac{u}{g}$

(ii) that the range of the particle on the plane is $\frac{u^2}{2g}$.

4. A light inextensible string passes over a smooth fixed pulley. It carries at one end a particle of mass 7 kg and at the other end a light, smooth pulley over which passes a light string with particles of mass 3 kg and 5 kg at its ends.



(i) On separate diagrams show the forces acting on each particle and on the movable pulley.

(ii) Find the accelerations of the three particles when the system is released from rest.

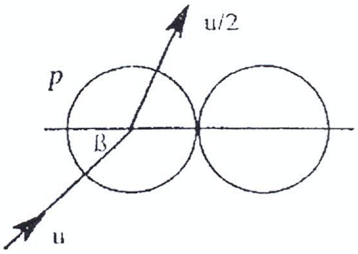
(iii) If the 3 kg mass is replaced by a mass of m kg, find the value of m if this particle does not move when the system is released from rest.

5. (a) Two smooth spheres of masses $2m$ and m moving in opposite directions with speeds u and $2u$, respectively, collide directly. If E_1 and E_2 are the sums of the kinetic energies of the two spheres before and after impact respectively, prove that

$$e = \sqrt{\frac{E_2}{E_1}}$$

where e is the coefficient of restitution.

(b) A smooth sphere P, moving with velocity u, impinges on an equal smooth sphere at rest, the direction of u just before impact being inclined at an angle β to the line of centres. If the speed of P after impact is $\frac{u}{2}$ and $\tan\beta = \frac{1}{2}$, show that the coefficient of restitution is also $\frac{1}{2}$.



1996

6. *(a)* A body of mass 10 kg moves with simple harmonic motion. At a displacement of 0.8 m from the centre of oscillation, the velocity and acceleration of the body are 2 m/s and 20 m/s² respectively.

Find

(i) the number of oscillations per second

(ii) the amplitude of motion

(iii) the maximum acceleration and hence show that the force to overcome the inertia of the body at the extremity of the oscillation is 223.6N.

(b) A light perfectly elastic string of natural length a and elastic constant k is fastened at one end p to a fixed point of a smooth horizontal table, and a particle of mass m is attached to the other end. The particle is held on the table at a distance $2a$ from p and then released.

Prove

(i) that the particle executes simple harmonic motion while the string is taut

(ii) that the particle reaches p after

$$\left(\frac{\pi}{2} + 1\right)\sqrt{\frac{m}{k}} \text{ seconds.}$$

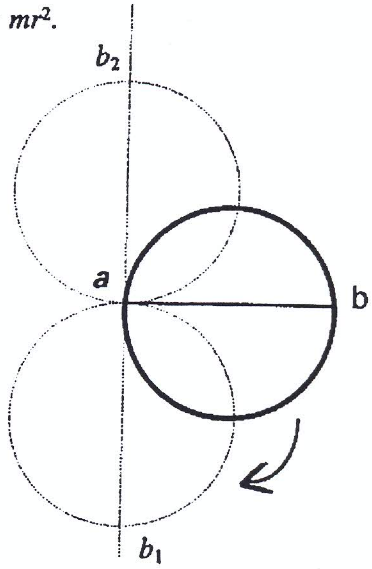
7. *(a)* A rod $[ac]$ of weight W and length 2 m has a particle of weight 2W fixed at a point on the rod 50 cm from a. The rod is kept at rest in a horizontal position by the action of three forces applied at a, b and c where b is the midpoint of $[ac]$. The force at b is four times the force at c.
Calculate the force at a in terms of W.

(b) Two uniform ladders $[ab]$ and $[ac]$, of equal length l and equal weight W, are smoothly jointed at a and stand with b and c in contact with a rough horizontal plane. The coefficient of friction at b and c is μ. If a person of weight W can stand anywhere on the ladders when b and c are a distance $2d$ apart, prove that

$$\mu \geq \frac{2d}{3\sqrt{l^2 - d^2}}.$$

8. *(a)* Prove that the moment of inertia of a uniform circular disc, of mass m and radius r, about an axis through its centre perpendicular to its plane is $\frac{1}{2}mr^2$.

(b) A uniform circular disc of radius r can move freely about a smooth pivot at a point a on its circumference. When its plane is vertical and the diameter $[ab]$ is horizontal the point b is given a velocity p vertically downwards.



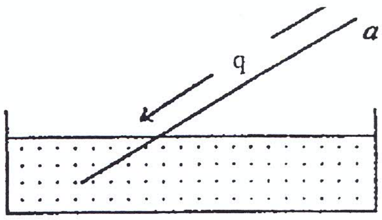
Find

(i) the angular velocity of the disc when b is vertically below a, i.e. at b_1

(ii) the value of p, in terms of r, if b just reaches the point where it is vertically above a, i.e. at b_2.

OVER→

9. *(a)* A uniform rod of length h and relative density s, is pivoted at one end a and is free to move about a horizontal axis. The other end of the rod is immersed in water. A length q of the rod remains above the surface of the water.



Show that $q^2 = h^2 (1 - s)$.

(b) When equal volumes of two substances are mixed the relative density of the mixture is 2.5. When equal weights of the same two substances are mixed the relative density of the mixture is 2.4.

Find the relative densities of the two substances.

10. *(a)* Solve the differential equation

$$\frac{dy}{dx} = 4y\cos x$$

if $y = e^2$ when $x = \frac{\pi}{6}$.

(b) A particle of mass m is projected vertically upwards with a velocity v of $\sqrt{\frac{2g}{k}}$, the air resistance being kv^2 per unit mass. Prove that

(i) the greatest height reached by the particle is $\frac{\ln 3}{2k}$

(ii) the velocity of the particle when passing through the point of projection on the way down is $\sqrt{\frac{2g}{3k}}$.

AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1995

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 23 JUNE – MORNING, 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9.8 m/s^2 .

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

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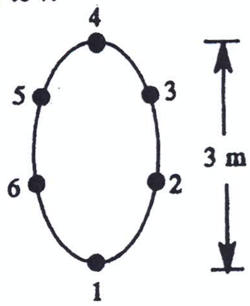
1. *(a)* A particle moving in a straight line with constant acceleration passes three points p, q, r and has speeds u and $7u$ at p and r respectively.

(i) Find its speed at q the mid-point of $[pr]$ in terms of u.
(ii) Show that the time from p to q is twice that from q to r.

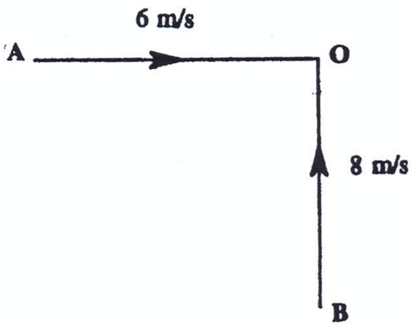
(b) A juggler throws up six balls, one after the other at equal intervals of time t, each to a height of 3 m. The first ball returns to his hand t seconds after the sixth was thrown up and is immediately thrown to the same height, and so on continually. (Assume that each ball moves vertically).



Find

(i) the initial velocity of each ball.
(ii) the time t.
(iii) the heights of the other balls when any one reaches the juggler's hand.

2. *(a)* Two particles A and B are moving along two perpendicular lines towards a point O with constant velocities of 6 m/s and 8 m/s respectively. When A is 64 m from O, B is 62 m from O.



(i) Find the distance of each particle from O after t seconds.

(ii) Hence, or otherwise, find the times at which their distance apart is 50 m.

(b) A girl wishes to swim across a river 60 m wide. The river flows with a velocity of q m/s parallel to the straight banks and the girl swims at a velocity of p m/s relative to the water. In crossing the river as quickly as possible she takes 100 s and is carried downstream 45 m.

Find

(i) p and q.
(ii) how long will it take her to swim in a straight line back to the original starting point.

3. *(a)* A ball kicked from a point p on level ground hit the ground for the first time 27 m from p after a time 3 s. The ball just passed over a wall standing 5.4 m from p.

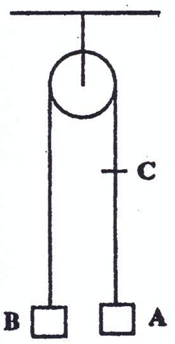
Find

(i) the horizontal and vertical components of its initial velocity.
(ii) the height of the wall.
(iii) the speed of the ball as it passed over the wall.

(b) A plane is inclined at an angle of 30° to the horizontal. A particle is projected up the plane with initial velocity 20 m/s at an angle θ to the plane. The plane of projection is vertical and contains the line of greatest slope. If the particle strikes the plane at right angles,

(i) find the angle of projection θ .
(ii) prove that if the angle of projection is increased to 45° then the particle strikes the plane obtusely and bounces back down the plane.

4. Two particles A and B of mass 0.4 kg and 0.5 kg respectively are connected by a light inextensible string which passes over a smooth pulley. When A has risen for 1 second, it passes a point C and picks up a mass of 0.2 kg.



Find

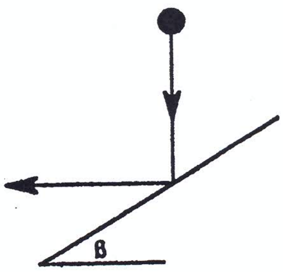
(i) the initial acceleration.
(ii) the velocity of A just before it picks up the mass at C.
(iii) using the principle of conservation of momentum, or otherwise, the velocity of A after picking up the mass at C.
(iv) the distance of A from C at the first position of instantaneous rest.

5. *(a)* Two smooth spheres of masses $2m$ and $3m$ respectively lie on a smooth horizontal table. The spheres are projected towards each other with speeds $4u$ and u respectively.

(i) Find the speed of each sphere after the collision in terms of e, the coefficient of restitution.

(ii) Show that the spheres will move in opposite directions after the collision if $e > \frac{1}{3}$.

(b) A ball falls vertically and strikes a smooth fixed plane. The plane is inclined at an angle β to the horizontal ($\beta < 45°$). The ball rebounds horizontally.



(i) Prove that $\tan \beta = \sqrt{e}$, where e is the coefficient of restitution.

(ii) Show that the fraction of kinetic energy lost during impact is $(1 - e)$.

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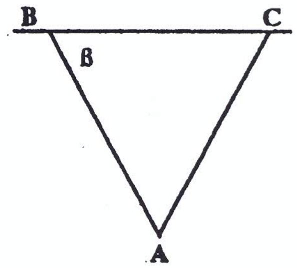
6. (a) A light string [op], of length l, is fixed at the end o, and is attached at the other end p to a particle which is moving uniformly in a horizontal circle whose centre is vertically below and distant h from o.

Prove that the period of the motion is $2\pi\sqrt{\frac{h}{g}}$.

(b) A particle of mass m, attached to a fixed point by a light inelastic string, describes a circle in a vertical plane. The tension of the string when the particle is at the highest point of the orbit is T_1 and when at the lowest point it is T_2. Prove that

$$T_2 = T_1 + 6mg.$$

7. Two uniform rods AB and AC of equal length and of weights 2W and W respectively are smoothly hinged together at A and hinged at B and C to a horizontal beam. The rods are in a vertical plane with A below BC.



Prove that

(i) the horizontal and vertical components of the reaction of the hinge at A on the rod AB are

$$\frac{3W}{4\tan\beta} \text{ and } \frac{W}{4}$$

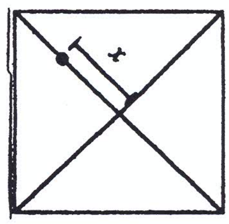
respectively, where ß is the angle ABC.

(ii) if the total reactions at B and C are perpendicular to each other, then

$$\tan \beta = \frac{3}{\sqrt{35}}.$$

8. (a) Prove that the moment of inertia of a uniform square lamina, of mass m and side $2a$, about an axis through its centre parallel to one of its sides is $\frac{1}{3}ma^2$.

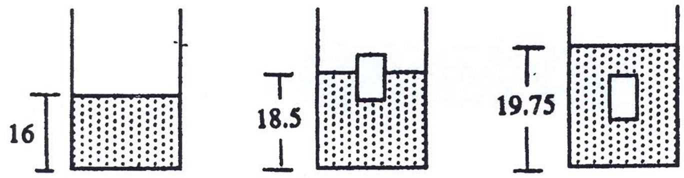
(b) A uniform square lamina of side $2a$ is freely pivoted at a point in one diagonal and oscillates in its own plane. Prove that when the period of small oscillations is a minimum the distance of the pivot from the centre is x where



$$3x^2 = 2a^2.$$

9. *(a)* What quantity of water must be mixed with 1 litre of milk to reduce its relative density from 1.03 to 1.02?

(b) A cylinder contains water to a height of 16 cm. A body of mass 0.02 kg is placed in the cylinder. It floats and the water level rises to 18.5 cm. The body is then completely submerged and the water level rises to 19.75 cm.



Find

(i) the relative density of the body.

(ii) the force required to submerge the body.

(iii) the volume of water in the cylinder.

10. *(a)* Solve the differential equation

$$(1 + x^2)\frac{dy}{dx} = \frac{4}{y}$$

if $x = 0$ when $y = 1$.

(b) A particle of mass m falls from rest against an air resistance of mkv, where k is constant and v is the speed. Prove that

(i) the time taken to acquire a speed of $\frac{g}{2k}$ is $\frac{\ln 2}{k}$.

(ii) the speed of the particle tends to a limit $\frac{g}{k}$.

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AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1994

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – MORNING, 9·30 to 12·00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s^2.

Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.

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1. *(a)* A lift, in a continuous descent, had uniform acceleration of 0.6 m/s^2 for the first part of its descent and a retardation of 0.8 m/s^2 for the remainder. The time, from rest to rest, was 14 seconds.

Draw a time-velocity graph and hence, or otherwise, find the distance descended.

(b) In a lift, moving upwards with acceleration f, a spring balance indicates an object to have a weight of 98 N. When the lift is moving downwards with acceleration 2f the weight appears to be 68.6 N.
Calculate
(i) the actual weight
(ii) the downward acceleration of the lift.

2. A cyclist A is pedalling at 3 m/s due east along a straight road. A second cyclist B is pedalling at 4 m/s due north along another straight road intersecting the first at a junction p.

(a) If A is 80 m and B is 40 m from p at a given moment, calculate

(i) the velocity of B relative to A.

(ii) how far each cyclist is from p when they are nearest together.

(b) If when A and B are 80 and 40 m from p, respectively, then A immediately accelerates at 0.1 m/s^2 and B decelerates at q m/s^2.

(i) Find the velocity of B relative to A in terms of time t.

(ii) Determine the value of q which causes them to arrive at p together.